Matrix Addition, Subtraction, and Scalar Multiplication

1. Tell whether the following matrices are equal or not equal.

a. $\begin{bmatrix} 2.5 & -1 \\ -0.2 & 3 \end{bmatrix}$ and $\begin{bmatrix} \frac{5}{2} & -1 \\ -\frac{1}{5} & 3 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$ and $\begin{bmatrix} 0 & -1 \\ 8 & 3 \end{bmatrix}$

2. If $\begin{bmatrix} 2x & 3 & 0 \\ 1 & 5 & 18 \\ 4 & z-7 & 8 \end{bmatrix} = \begin{bmatrix} 17 & 3 & 0 \\ 1 & 5 & -3y+4 \\ 4 & 11 & 8 \end{bmatrix}$, find x, y, and z.

3. Perform the indicated operation(s).

a. $6\begin{bmatrix} 2 & -3 \\ 8 & 4 \end{bmatrix}$

b. $-1\begin{bmatrix} 4 & -7 \\ 3 & 3 \\ 2 & -9 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ 9 & -8 \\ 1 & -4 \end{bmatrix}$

4. Perform the indicated operation, if possible.

a. $\begin{bmatrix} 0 \\ 4 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ -9 \\ 8 \end{bmatrix}$

b. $\begin{bmatrix} 3 & 3 \\ 9 & -5 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ -3 & 5 \end{bmatrix}$

c. $\begin{bmatrix} 9 & -7 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ 4 & -8 \end{bmatrix}$

5. Solve the matrix equation for x and y.

$$4\left(\begin{bmatrix} 8 & 0 \\ -1 & 2y \end{bmatrix} + \begin{bmatrix} 4 & -2x \\ 1 & 6 \end{bmatrix}\right) = \begin{bmatrix} 48 & -48 \\ 0 & 8 \end{bmatrix}$$

6. Solve the matrix equation for x and y.

$$3\left(\begin{bmatrix} 10 & 2 \\ 5 & 4y \end{bmatrix} - \begin{bmatrix} x & 5 \\ -1 & 1 \end{bmatrix}\right) = \begin{bmatrix} 0 & -9 \\ 18 & 21 \end{bmatrix}$$

7. A wholesale nursery does business with garden departments of two major supermarket chains. Sales of different types of plants are given for the months of November and April.

$$\text{November} \qquad \begin{array}{l} \\ \text{Shrubs} \\ \text{Perennials} \\ \text{Houseplants} \\ \text{Vegetables} \end{array} \begin{array}{c} \begin{array}{cc} \text{Fred Myers} & \text{Home Club} \end{array} \\ \begin{bmatrix} 2300 & 3100 \\ 1200 & 800 \\ 15{,}000 & 17{,}050 \\ 300 & 100 \end{bmatrix} \end{array} \qquad \text{April} \qquad \begin{array}{l} \\ \text{Shrubs} \\ \text{Perennials} \\ \text{Houseplants} \\ \text{Vegetables} \end{array} \begin{array}{c} \begin{array}{cc} \text{Fred Myers} & \text{Home Club} \end{array} \\ \begin{bmatrix} 8000 & 7800 \\ 12{,}000 & 13{,}700 \\ 6000 & 7500 \\ 10{,}000 & 9500 \end{bmatrix} \end{array}$$

a. Calculate the change in sales from November to April for each store for each type of plant and express your answer in a matrix with labels on the rows and columns.

b. Due to a surging interest in gardening, it is projected that plant sales in the spring will increase by 15%. With this increase, how many of each plant type will each store sell next April?

8. Use matrices to organize the following information about car insurance rates. Then use the matrices to write a matrix that shows the monthly changes in car insurance payments from this year to next year.

This year *For 1 car, comprehensive, collision, and basic insurance cost $612.15, $518.29, and $486.91.*
For 2 cars ,comprehensive, collision, and basic insurance cost $1150.32, $984.16, and $892.51.
Next year *For 1 car, comprehensive, collision, and basic insurance cost $616.28, $520.39, and $490.05.*
For 2 cars, comprehensive, collision, and basic insurance cost $1155.84, $987.72, and $895.13.

8. Condominium owners must pay yearly fees to cover the cost of maintenance, landscaping, and remodeling. The fees this year are $96, $18, and $66 for a 1-bedroom unit, and $128, $24, and $88 for a 2-bedroom unit. The fees next year are $105, $20, and $73 for a 1-bedroom unit, and $141, $26, and $97 for a 2-bedroom unit. Use matrices to organize the information. Then use the matrices to find the monthly changes in fees from this year to next year.

LESSON 4.1

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Reteaching with Practice

For use with pages 199–206

GOAL **Add and subtract matrices, multiply a matrix by a scalar, solve matrix equations, and use matrices in real-life situations**

VOCABULARY

A **matrix** is a rectangular arrangement of numbers in rows and columns where the numbers are called **entries.**

The **dimensions** of a matrix are given as *the number of rows* $\times$ *the number of columns*.

Scalar multiplication is the process of multiplying each entry in a matrix by a **scalar,** a real number.

Lesson 4.1

EXAMPLE 1 ## *Adding and Subtracting Matrices*

Perform the indicated operation, if possible.

a. $\begin{bmatrix} 2 \\ -1 \\ 7 \end{bmatrix} + \begin{bmatrix} 4 & 0 & -6 \end{bmatrix}$

b. $\begin{bmatrix} 2 & 3 & -5 \\ -1 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 3 \\ 3 & -2 & -1 \end{bmatrix}$

SOLUTION

a. To add or subtract matrices, they must have the same dimensions.

Since $\begin{bmatrix} 2 \\ -1 \\ 7 \end{bmatrix}$ is a 3×1 matrix and $\begin{bmatrix} 4 & 0 & -6 \end{bmatrix}$ is a 1×3 matrix, you cannot add them.

b. Since both matrices are 2×3, you can subtract them.

$$\begin{bmatrix} 2 & 3 & -5 \\ -1 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 3 \\ 3 & -2 & -1 \end{bmatrix} = \begin{bmatrix} 2-0 & 3-1 & -5-3 \\ -1-3 & 0-(-2) & 4-(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & -8 \\ -4 & 2 & 5 \end{bmatrix}$$

Exercises for Example 1

Perform the indicated operation, if possible.

1. $\begin{bmatrix} 2 & 5 \\ -1 & -3 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ 0 & 7 \\ 0 & 0 \end{bmatrix}$

2. $\begin{bmatrix} 5 & 0 \\ -2 & 1 \\ 4 & -3 \end{bmatrix} - \begin{bmatrix} 4 & -6 \\ 2 & -2 \\ -1 & 3 \end{bmatrix}$

EXAMPLE 2 ## *Solving a Matrix Equation*

Solve the matrix equation for x and y: $-2x\begin{bmatrix} 5 & 0 & -1 \\ 4 & 2 & 8 \end{bmatrix} = \begin{bmatrix} 20 & 0 & -4 \\ 16 & 8 & y \end{bmatrix}$.

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Reteaching with Practice

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SOLUTION

$$-2x\begin{bmatrix} 5 & 0 & -1 \\ 4 & 2 & 8 \end{bmatrix} = \begin{bmatrix} 20 & 0 & -4 \\ 16 & 8 & y \end{bmatrix}$$ Write original equation.

$$\begin{bmatrix} -10x & 0 & 2x \\ -8x & -4x & -16x \end{bmatrix} = \begin{bmatrix} 20 & 0 & -4 \\ 16 & 8 & y \end{bmatrix}$$ Multiply by $-2x$.

$-10x = 20$ $\quad -16x = y$ Equate corresponding entries involving x and y.

$x = -2$ $\quad -16(-2) = y$ Solve the resulting equations.

$32 = y$

Exercises for Example 2

Solve the matrix equation for *x* and *y*.

3. $\begin{bmatrix} 3x & 2 & 4 \end{bmatrix} = \begin{bmatrix} 9 & -y & 4 \end{bmatrix}$

4. $\begin{bmatrix} 2y & -1 \\ -6 & 0 \end{bmatrix} + \begin{bmatrix} 5 & 4 \\ x & 8 \end{bmatrix} = \begin{bmatrix} -5 & 3 \\ -7 & 8 \end{bmatrix}$

EXAMPLE 3

Using Matrix Operations

Write a matrix that shows the average costs in health care from this year to next year.

This Year (A)

$$\begin{array}{r} \\ \text{Comprehensive} \\ \text{HMO Standard} \\ \text{HMO Plus} \end{array} \begin{array}{c} \begin{array}{cc} \text{Individual} & \text{Family} \end{array} \\ \begin{bmatrix} \$694.32 & \$1725.36 \\ \$451.80 & \$1187.76 \\ \$489.48 & \$1248.12 \end{bmatrix} \end{array}$$

Next Year (B)

$$\begin{array}{r} \\ \text{Comprehensive} \\ \text{HMO Standard} \\ \text{HMO Plus} \end{array} \begin{array}{c} \begin{array}{cc} \text{Individual} & \text{Family} \end{array} \\ \begin{bmatrix} \$683.91 & \$1699.48 \\ \$463.10 & \$1217.45 \\ \$499.27 & \$1273.08 \end{bmatrix} \end{array}$$

SOLUTION

Begin by adding matrix A and matrix B to determine the total costs for two years. Then multiply the result by $\frac{1}{2}$, which is equivalent to dividing by 2. Round your answers to the nearest cent to find the average.

$$\frac{1}{2}(A + B) = \frac{1}{2}\left(\begin{bmatrix} 694.32 & 1725.36 \\ 451.80 & 1187.76 \\ 489.48 & 1248.12 \end{bmatrix} + \begin{bmatrix} 683.91 & 1699.48 \\ 463.10 & 1217.45 \\ 499.27 & 1273.08 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 1378.23 & 3424.84 \\ 914.90 & 2405.21 \\ 988.75 & 2521.20 \end{bmatrix} = \begin{bmatrix} \$689.12 & \$1712.42 \\ \$457.45 & \$1202.61 \\ \$494.38 & \$1260.60 \end{bmatrix}$$

Exercises for Example 3

5. Using the matrix B on health care costs, write a matrix C for the following year that shows the costs after a 2% decrease.

6. Write a matrix which will show the monthly payment following a 3% increase in the costs from matrix B.

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Practice A

For use with pages 199–206

Determine the dimensions of the matrix.

1. $\begin{bmatrix} 3 & 5 & -7 \\ 1 & 2 & 9 \\ -2 & 6 & 1 \\ 4 & -3 & 5 \end{bmatrix}$ **2.** $\begin{bmatrix} 4 & 9 \\ -5 & 1 \\ 2 & -6 \end{bmatrix}$ **3.** $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$ **4.** $\begin{bmatrix} 1 & 4 & 5 & -2 \\ -6 & 2 & 0 & 3 \\ 3 & 8 & -1 & 4 \end{bmatrix}$

Tell whether the matrices are *equal* or *not equal.*

5. $\begin{bmatrix} 3 & 4 \\ -7 & 1 \end{bmatrix}, \begin{bmatrix} 3 & 4 \\ 7 & -1 \end{bmatrix}$ **6.** $\begin{bmatrix} 2 & -1 & 6 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$ **7.** $\begin{bmatrix} 1 & 0 \\ 4 & -3 \end{bmatrix}, \begin{bmatrix} \frac{2}{2} & 0 \\ \frac{8}{2} & -\frac{3}{1} \end{bmatrix}$

Perform the indicated operation, if possible. If not possible, state the reason.

8. $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$ **9.** $\begin{bmatrix} 2 \\ 5 \end{bmatrix} + \begin{bmatrix} -3 \\ 1 \end{bmatrix}$ **10.** $\begin{bmatrix} 4 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -4 \end{bmatrix}$

11. $\begin{bmatrix} 2 \\ -7 \end{bmatrix} + \begin{bmatrix} -3 & 4 \end{bmatrix}$ **12.** $\begin{bmatrix} 0 & 4 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$ **13.** $\begin{bmatrix} 3 \\ -4 \end{bmatrix} - \begin{bmatrix} 4 \\ 7 \end{bmatrix}$

14. $\begin{bmatrix} 1 & 4 \\ -5 & 8 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ -5 & 8 \end{bmatrix}$ **15.** $\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 5 \end{bmatrix}$ **16.** $\begin{bmatrix} 0 & 0 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} -13 & 2 \\ 1 & -7 \end{bmatrix}$

Perform the indicated operation.

17. $2\begin{bmatrix} 1 & 6 \\ -3 & 2 \end{bmatrix}$ **18.** $-3\begin{bmatrix} 1 & 0 \\ -3 & 6 \end{bmatrix}$ **19.** $5\begin{bmatrix} 2 \\ -5 \end{bmatrix}$

20. $-4\begin{bmatrix} -3 & 6 & 1 \end{bmatrix}$ **21.** $8\begin{bmatrix} 3 \\ 0 \\ -5 \end{bmatrix}$ **22.** $-1\begin{bmatrix} 2 & 5 & -3 \\ 6 & -1 & -7 \\ 0 & 0 & 9 \end{bmatrix}$

Solve the matrix for *x* and *y*.

23. $\begin{bmatrix} x & 3 \\ 5 & y \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 5 & -4 \end{bmatrix}$ **24.** $\begin{bmatrix} 2x \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 10 \\ 3 \\ 4y \end{bmatrix}$ **25.** $\begin{bmatrix} 3x & -21 \end{bmatrix} = \begin{bmatrix} 21 & 7y \end{bmatrix}$

26. ***Endangered and Threatened Species*** The matrices below show the number of endangered and threatened animal and plant species as of June 30, 1996. Use matrix addition to find the total number of endangered and threatened species. *(Source: 1997 Information Please Almanac)*

ENDANGERED $\begin{array}{r c} & \begin{array}{cc} \textbf{U.S.} & \textbf{Foreign} \end{array} \\ \textbf{Animal} & \multirow{2}{*}{} \\ \end{array}$

$$\text{ENDANGERED: } \begin{array}{r} \\ \textbf{Animal} \\ \textbf{Plant} \end{array} \begin{array}{c} \begin{array}{cc} \textbf{U.S.} & \textbf{Foreign} \end{array} \\ \begin{bmatrix} 320 & 521 \\ 431 & 1 \end{bmatrix} \end{array} \qquad \text{THREATENED: } \begin{array}{r} \\ \textbf{Animal} \\ \textbf{Plant} \end{array} \begin{array}{c} \begin{array}{cc} \textbf{U.S.} & \textbf{Foreign} \end{array} \\ \begin{bmatrix} 115 & 41 \\ 94 & 2 \end{bmatrix} \end{array}$$

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Practice B

For use with pages 199–206

Perform the indicated operation, if possible. If not possible, state the reason.

1. $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} -5 & 2 \\ 7 & 1 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 3 \\ -2 & 5 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 9 \\ 7 & 1 \\ -2 & 6 \end{bmatrix}$

3. $\begin{bmatrix} 4 & 3 & -2 \\ 1 & 5 & 4 \\ 2 & 7 & 6 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 5 \\ 4 & -1 & 3 \\ 6 & 7 & 9 \end{bmatrix}$

4. $\begin{bmatrix} -3 & 0 & 10 & -8 \end{bmatrix} + \begin{bmatrix} -1 \\ 7 \\ -11 \end{bmatrix}$

5. $\begin{bmatrix} -8 & 3 & 9 \\ 4 & 12 & -1 \\ -4 & -6 & 8 \end{bmatrix} + \begin{bmatrix} -4 & -1 & 6 \\ 12 & -12 & 10 \\ -5 & -7 & -11 \end{bmatrix}$

6. $\begin{bmatrix} 7 & -2 & -5 \\ -1 & -7 & 3 \\ 8 & -10 & -13 \end{bmatrix} - \begin{bmatrix} -1 & -3 & 4 \\ -10 & -11 & 8 \\ 8 & -10 & -13 \end{bmatrix}$

7. $\begin{bmatrix} 6 & -2 & 1 & 8 \end{bmatrix} + \begin{bmatrix} 7 & 5 & -3 & 2 \end{bmatrix}$

Perform the indicated operation.

8. $3\begin{bmatrix} 1 & 4 \\ -3 & 2 \end{bmatrix}$

9. $-2\begin{bmatrix} -\frac{1}{2} & 0 & 2 \\ 3 & 4 & -1 \\ -2 & \frac{3}{2} & 5 \end{bmatrix}$

10. $-5\begin{bmatrix} 1 & 4 & -2 & 3 \\ 0 & -5 & 1 & 4 \end{bmatrix}$

Perform the indicated operations.

11. $\left(\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} - \begin{bmatrix} -3 & -4 \\ 2 & 5 \end{bmatrix}\right) + \begin{bmatrix} 2 & 5 \\ 3 & 9 \end{bmatrix}$

12. $\begin{bmatrix} 1 & 3 \\ 2 & 0 \\ 5 & 6 \end{bmatrix} - \left(\begin{bmatrix} 9 & 2 \\ 4 & -1 \\ -3 & 1 \end{bmatrix} + \begin{bmatrix} -7 & 3 \\ 0 & 1 \\ 5 & 3 \end{bmatrix}\right)$

13. $3\begin{bmatrix} 1 & -4 \\ 3 & 8 \end{bmatrix} + 5\begin{bmatrix} -1 & 0 \\ 2 & -4 \end{bmatrix}$

14. $2\left(\begin{bmatrix} 2 & 8 & -1 \\ 0 & 2 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 6 & 3 \\ -1 & -1 & 3 \end{bmatrix}\right)$

Health Club Membership **In Exercises 15 and 16, use the following information.**

A health club offers three different membership plans. With Plan A, you can use all club facilities: the pool, fitness center, and racket club. With Plan B, you can use the pool and fitness center. With Plan C, you can only use the racket club facilities. The matrices below show the annual cost for a Single and a Family membership for the years 1998 through 2000.

1998

$$\begin{array}{r} \\ \textbf{Plan A} \\ \textbf{Plan B} \\ \textbf{Plan C} \end{array} \begin{array}{c} \begin{array}{cc} \textbf{Single} & \textbf{Family} \end{array} \\ \begin{bmatrix} 336 & 624 \\ 228 & 528 \\ 216 & 385 \end{bmatrix} \end{array}$$

1999

$$\begin{array}{r} \\ \textbf{Plan A} \\ \textbf{Plan B} \\ \textbf{Plan C} \end{array} \begin{array}{c} \begin{array}{cc} \textbf{Single} & \textbf{Family} \end{array} \\ \begin{bmatrix} 384 & 720 \\ 312 & 576 \\ 240 & 432 \end{bmatrix} \end{array}$$

2000

$$\begin{array}{r} \\ \textbf{Plan A} \\ \textbf{Plan B} \\ \textbf{Plan C} \end{array} \begin{array}{c} \begin{array}{cc} \textbf{Single} & \textbf{Family} \end{array} \\ \begin{bmatrix} 420 & 792 \\ 360 & 672 \\ 288 & 528 \end{bmatrix} \end{array}$$

15. You purchased a Single Plan A membership in 1998, a Family Plan B membership in 1999, and a Family Plan A Membership in 2000? How much did you spend for your membership over the three years?

16. You purchased a Family Plan C membership in 1998, and upgraded to the next highest plan each year. How much did you spend for your membership over the three years?

Matrix Multiplication

1. State whether AB is defined. If so, give the dimensions.

   a. A: 2×4, B: 4×3

   b. A: 1×4, B: 1×4

2. If $\begin{bmatrix} 2 & 8 \\ -3 & 1 \\ 7 & 6 \end{bmatrix} \times J = \begin{bmatrix} -10 & 4 & 30 & 34 \\ -11 & -6 & 7 & -12 \\ 9 & 14 & 17 & 53 \end{bmatrix}$, what are the dimensions of matrix J?

3. Find AB if $A = \begin{bmatrix} -1 & 5 \\ 5 & 2 \\ 0 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -3 \\ 6 & 8 \end{bmatrix}$.

4. $A = \begin{bmatrix} 4 & 1 \\ 0 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & -3 \\ 1 & 2 \end{bmatrix}$

   a. Find AB.

   b. Find BA.

5. Find AB if $A = \begin{bmatrix} 8 & 0 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 6 \\ -3 \end{bmatrix}$

6. $A = \begin{bmatrix} 2 & -2 \\ 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix}$, and $C = \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$

a. Find $B(A + C)$.

b. Find $BA + BC$.

7. Solve for x and y. $\begin{bmatrix} 1 & x \\ 2 & y \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$

8. The average number of gallons of gas sold daily at three different gas stations is given in the matrix A below. The selling price per gallon for each grade of gas is given in the matrix B below. Find the total sales for each gas station.

$$A = \begin{array}{c} \\ \textit{Unleaded} \\ \textit{Plus} \\ \textit{Super} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Townco} & \textit{Star} & \textit{Isico} \end{array} \\ \begin{bmatrix} 3000 & 2500 & 2750 \\ 480 & 310 & 350 \\ 510 & 410 & 380 \end{bmatrix} \end{array} \qquad B = \begin{array}{c} \\ \textit{Unleaded} \\ \textit{Plus} \\ \textit{Super} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Townco} & \textit{Star} & \textit{Isico} \end{array} \\ \begin{bmatrix} 1.96 & 1.99 & 1.94 \\ 2.04 & 2.08 & 2.06 \\ 2.10 & 2.12 & 2.11 \end{bmatrix} \end{array}$$

9. A flu epidemic occurs in Middletown schools and each student is susceptible, sick, or infected. The percentage of students in each category by grade level and the population distribution of the Middletown school district is given below. How many sick girls are there?

$$\begin{array}{c} \\ \text{Elementary} \\ \text{Middle School} \\ \text{High School} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Susceptible} & \text{Sick} & \text{Infected} \end{array} \\ \begin{bmatrix} 60\% & 25\% & 15\% \\ 65\% & 15\% & 20\% \\ 70\% & 10\% & 20\% \end{bmatrix} \end{array} \qquad \begin{array}{c} \\ \text{Elementary} \\ \text{Middle School} \\ \text{High School} \end{array} \begin{array}{c} \begin{array}{cc} \text{Girls} & \text{Boys} \end{array} \\ \begin{bmatrix} 1610 & 1580 \\ 830 & 795 \\ 1115 & 1100 \end{bmatrix} \end{array}$$

10. A contractor is building homes according to three plans, A, B, and C. All of the plans use the same kind of windows and doors.

In Plan A, the house has 12 windows and 4 exterior doors.
Plan B uses 20 windows and 5 exterior doors.
Plan C uses 10 windows and 3 exterior doors.
The costs of a window and a door are \$90 and \$185, respectively.

a. Write a row matrix *C* to represent the cost of the windows and doors.
b. Write a matrix *H* to represent the needs for windows and doors for each model home
c. Find *CH.*
d. What is the interpretation of the first entry in the first row of matrix *CH*?

11. The builder plans to build a number of houses using each model plan. He plans 25 of model A, 18 of model B, and 32 of model C.

a. Write a 3x1 matrix *D* to represent this information.
b. Find the product matrix *HD.* What does each entry represent?
c. What will be the total cost for all windows and doors?

12. Two lacrosse teams submit equipment lists to their sponsors. Use matrix multiplication to find the total cost of the equipment for each team.

Women's team: 5 sticks, 15 balls, and 16 uniforms
Men's team: 8 sticks, 22 balls, and 17 uniforms
Each stick costs \$55, each ball costs \$6, and each uniform costs \$35.

LESSON 4.2

NAME ______________________ DATE __________

Reteaching with Practice

For use with pages 208–213

GOAL **Multiply two matrices**

VOCABULARY

The product of two matrices A and B is defined only if the number of columns in A is equal to the number of rows in B.

If A is an $m \times n$ matrix and B is an $n \times p$ matrix, then the product AB is an $m \times p$ matrix.

EXAMPLE 1 ***Finding the Product of Two Matrices***

Find AB and BA if $A = \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 & -3 & 1 \end{bmatrix}$.

SOLUTION

$$AB = \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix}\begin{bmatrix} 4 & 3 & -3 & 1 \end{bmatrix}$$

Because the number of columns in A equals the number of rows in B, the product AB will be a 3×4 matrix.

$$= \begin{bmatrix} 2(4) & 2(3) & 2(-3) & 2(1) \\ -2(4) & -2(3) & -2(-3) & -2(1) \\ 0(4) & 0(3) & 0(-3) & 0(1) \end{bmatrix}$$

Multiply the entries in the first row of A by the entries in the first column of B.

$$= \begin{bmatrix} 8 & 6 & -6 & 2 \\ -8 & -6 & 6 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

BA is undefined because B is a 1×4 matrix and A is a 3×1 matrix. The number of columns in B does not equal the number of rows in A.

Exercises for Example 1

Find the product. If it is not defined, state the reason.

1. $\begin{bmatrix} 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} -1 & 4 \\ 0 & 1 \\ 5 & 2 \end{bmatrix}$

2. $\begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} -1 & 5 \\ 6 & 2 \end{bmatrix}$

3. $\begin{bmatrix} 2 & -1 & 0 \\ 0 & 5 & 3 \\ 1 & -2 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 2 & 3 & 3 \end{bmatrix}$

4. $\begin{bmatrix} 0 & 1 \\ 4 & 3 \\ 5 & -1 \end{bmatrix}\begin{bmatrix} 2 & -1 & 0 \\ 3 & 4 & 1 \end{bmatrix}$

Lesson 4.2

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Reteaching with Practice

For use with pages 208–213

EXAMPLE 2 Using Matrix Operations

If $A = \begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 5 \\ 2 & 1 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$, simplify each expression.

a. $A(BC)$

b. $(AB)C$

SOLUTION

a. $A(BC) = \begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix}\left(\begin{bmatrix} -3 & 5 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}\right)$ Substitute the matrices for A, B, and C.

$= \begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -3 & 1 \\ 2 & -5 \end{bmatrix}$ Multiply B by C first.

$= \begin{bmatrix} -6 & -11 \\ 3 & -1 \end{bmatrix}$ Multiply A by the result.

b. $(AB)C = \left(\begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -3 & 5 \\ 2 & 1 \end{bmatrix}\right)\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$ Substitute the matrices for A, B, and C.

$= \begin{bmatrix} -6 & 23 \\ 3 & -5 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$ Multiply A by B first.

$= \begin{bmatrix} -6 & -11 \\ 3 & -1 \end{bmatrix}$ Multiply the result by C.

Exercises for Example 2

Use the given matrices to simplify the expression.

$A = \begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -3 \\ 0 & 2 \end{bmatrix}$, $C = \begin{bmatrix} -2 & -3 \\ 1 & 1 \end{bmatrix}$

5. AA

6. $A(B + C)$

7. $2(CA)$

8. $AB + BC$

9. $A(B - C)$

10. $-2(BA)$

NAME ______________________________ DATE __________

Practice A

For use with pages 208–213

State whether the product *AB* is defined. If so, give the dimensions of *AB*.

1. $A: 2 \times 2,\ B: 3 \times 2$

2. $A: 3 \times 4,\ B: 4 \times 3$

3. $A: 2 \times 5,\ B: 5 \times 1$

4. $A: 3 \times 2,\ B: 2 \times 2$

5. $A: 4 \times 1,\ B: 4 \times 1$

6. $A: 3 \times 4,\ B: 4 \times 5$

7. $A: 3 \times 5,\ B: 3 \times 3$

8. $A: 2 \times 4,\ B: 4 \times 4$

9. $A: 1 \times 6,\ B: 6 \times 1$

Complete the next step of the matrix multiplication.

10. $\begin{bmatrix} 3 & 1 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 2 & 1 & 0 \\ 3 & -2 & 4 \end{bmatrix} = \begin{bmatrix} (3)(2) + (1)(3) & (3)(1) + (1)(-2) & (3)(0) + (1)(4) \\ ? & ? & ? \end{bmatrix}$

11. $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}\begin{bmatrix} -4 & 6 \end{bmatrix} = \begin{bmatrix} 1(-4) & 1(6) \\ ? & ? \\ ? & ? \end{bmatrix}$

12. $\begin{bmatrix} 2 & 1 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} -2 & -1 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} (2)(-2) + (1)(1) & ? \\ (0)(-2) + (-3)(1) & ? \end{bmatrix}$

Find the product. If it is not defined, state the reason.

13. $\begin{bmatrix} 2 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \end{bmatrix}$

14. $\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

15. $\begin{bmatrix} 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$

16. $\begin{bmatrix} 3 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & -4 \end{bmatrix}$

17. $\begin{bmatrix} -1 \\ -2 \end{bmatrix}\begin{bmatrix} 2 & -3 \end{bmatrix}$

18. $\begin{bmatrix} 2 \\ 3 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix}$

19. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & 3 \\ 2 & 5 \end{bmatrix}$

20. $\begin{bmatrix} 1 & 2 & -4 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$

21. $\begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

22. ***Senior Play*** The senior class play was performed on three different evenings. The attendance for each evening is shown in the table below. Adult tickets sold for \$3.50. Student tickets sold for \$2.50. Use matrix multiplication to determine how much money was taken in each night.

Performance	*Adults*	*Students*
Opening night	420	300
Second night	400	450
Final night	510	475

NAME ______________________________ DATE __________

Practice B

For use with pages 208–213

State the dimensions of each matrix and determine whether the product AB is defined. If it is, state the dimensions of AB.

1. $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \\ 4 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 9 & -3 \end{bmatrix}$

2. $A = \begin{bmatrix} 1 & 4 & -2 \\ 3 & -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 & -2 \\ 5 & 2 & 4 \\ -3 & -6 & 7 \end{bmatrix}$

3. $A = \begin{bmatrix} 5 \\ -2 \\ 3 \\ 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 7 \end{bmatrix}$

4. $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \\ -1 & 6 \\ 0 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 7 & 3 & -4 \\ 6 & -2 & 6 & 1 \\ 5 & 4 & 8 & 0 \end{bmatrix}$

Find the product. If it is not defined, state the reason.

5. $\begin{bmatrix} -3 & 5 \end{bmatrix}\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

6. $\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

7. $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & -3 \end{bmatrix}$

8. $\begin{bmatrix} 1 & -2 \\ 6 & 4 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$

9. $\begin{bmatrix} 2 & 0 & 1 \\ -3 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}\begin{bmatrix} -2 & -1 & 2 \\ 1 & 0 & 3 \\ 0 & -4 & 1 \end{bmatrix}$

10. $\begin{bmatrix} -1 & 3 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 1 & 3 & 4 \\ -2 & 0 & 5 \end{bmatrix}$

11. $\begin{bmatrix} -3 & 4 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ -5 \\ 3 \end{bmatrix}$

12. $\begin{bmatrix} 1 & -2 \\ 3 & 0 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 2 & -1 & 4 & 2 \\ 1 & 0 & 5 & -3 \end{bmatrix}$

13. $\begin{bmatrix} 4 & -1 \\ 0 & 2 \\ 3 & 1 \\ 6 & -2 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix}$

Simplify the expression.

14. $4\begin{bmatrix} \frac{1}{2} & -1 \\ -\frac{3}{4} & \frac{3}{2} \end{bmatrix}\begin{bmatrix} -3 & -5 \\ 4 & 2 \end{bmatrix}$

15. $\begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} 5 & 2 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} -3 & 5 \\ 2 & -2 \end{bmatrix}$

16. $\begin{bmatrix} 2 & -4 & 0 \\ 0 & 3 & 6 \\ -1 & 5 & 1 \end{bmatrix}\left(\begin{bmatrix} 1 & 2 \\ -3 & 0 \\ 5 & 1 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ 0 & 2 \\ 4 & 5 \end{bmatrix}\right)$

17. $\left(\begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}\right)\begin{bmatrix} 4 & 4 \end{bmatrix}$

18. ***Senior Play*** The senior class play was performed on three different evenings. The attendance for each evening is shown in the table below. Adult tickets sold for \$3.50. Student tickets sold for \$2.50. Use matrix multiplication to determine how much money was taken in each night.

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